# Noise in satellite links

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Considering the difficulties achieving a terrestrial link, it might be surprising that satellite links, covering much greater distance are possible at all. One of the most important factors to explain this is the noise involved. When signals originate at the satellite, they are virtually free of noise. The origin of noise and the meaning of the noise figure and temperature in relation to receivers will be explained. The effect of the Earth atmosphere on signal-to-noise ratio (SNR) will be illustrated with a real-life example.

## 1 Theory

### 1.1 Thermal noise

Except at absolute zero temperature, the electrons in every conductor (resistor) are always in thermal motion, resulting in a voltage difference between the resistor's terminals. This can be modeled as a noise source  $N_{th}$  inside the resistor, as shown in figure 1. If the resistor



Figure 1: a non-ideal resistor and its equivalent scheme, attached to an amplifier

is connected to a matched 'load', e.g. the input impedance of an LNA (low noise amplifier), the thermal noise source in the resistor delivers a power to the load, equal to:

$$N = kTB,\tag{1}$$

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with k the Bolzmann constant 
$$(1, 38.10^{-23} \frac{W}{K.Hz})$$
,  
T the absolute temperature in Kelvin, and  
B the bandwidth [Hz].

Note that the thermal noise power is independent of the resistance value, but proportional to the resistor temperature. Thus, if one would connect a resistor (dummy load) to a sensitive power meter, one would notice an increase in noise power when the resistor is heated by e.g. a candle flame!

Because of the simple relation between the noise and the resistor temperature, it is sensible to define an *effective noise temperature* for other noise sources too, even if they are not thermal in origin, e.g. interference from another station. A noise source having a 'noise temperature  $T_n$ ' generates a noise power, equal to the thermal noise that would be generated by a resistor at temperature  $T_n$ .

It may come as a surprise that resistors exhibit a noisy voltage at their terminals. A very comparable phenomenon exists for *matter*, or bodies at a certain temperature: every object or *matter* that absorbs radiation also produces noise through radiation. Again, this may sound strange at first sight, but after all it is just an extension to 'low' frequencies, of the common observation that hot objects (such as a candle flame) emit visible light. For a perfect absorbing object or so-called *blackbody* the emanating noise is proportional to the object's temperature, and is also given by equation 1. Most of the time, this relation is also a good approximation for non-perfectly absorbing objects, such as the Sun for instance. Imagine one would point an antenna towards the Sun, such that it is the only object contained in the antenna bundle. If a power meter is attached to the antenna (as in figure 2), and one would look at the power contained in a bandwidth of e.g. 4kHz, then the readout would be equal to  $1, 38.10^{-23}.10^{5}.4000 = 5, 5^{-15}$ Watts = -113 dBm, because the Sun is approximately at  $10^{5}$  Kelvin. In case the antenna isn't directional enough, and the bundle is too wide, the power measured will be a weighted average of the different bodies the antenna is looking at.



Figure 2: Measuring the thermal noise emerging from the Sun

### **1.2** Noise figure and noise temperature

Consider an non-ideal amplifier with gain G, having an input consisting of a useful signal  $S_{in}$  with some noise  $N_{in}$ . Of course, the signal as well as the noise will be amplified. Moreover, the amplifier will add some extra noise to the signal. Because it is a hard to locate this strange noise source in the amplifier, it can be replaced with an *ideal* amplifier and an artificial noise source  $N_{ai}$  at its input, accounting for the noise, induced by the amplifier. In figure 3, such a setup is shown.



Figure 3: a non-ideal amplifier and its equivalent scheme

#### 1.2.1 Noise figure

The noise figure F indicates the decrease in signal-to-noise ratio (SNR) between the input and the output of a system. In other words,

$$F = \frac{SNR_{in}}{SNR_{out}} = \frac{\left(\frac{S_{in}}{N_{in}}\right)}{\left(\frac{GS_{in}}{G(N_{in}+N_{ai})}\right)} = \frac{N_{in}+N_{ai}}{N_{in}} = 1 + \frac{N_{ai}}{N_{in}}.$$
(2)

Judging from equation (2), the noise figure is dependent on the noise power at the input  $N_{in}$ . To make it possible to compare different amplifiers, people have to a fixed reference noise power, equal to the noise generated in a resistor at  $T_0 = 290K$ . The noise power of such a source at 290K is equal to:

$$N_0 = kT_0 = 1,38.10^{-23}.290 = 4.10^{-23}$$
W/Hz

In practice this means that an 'amplifier having a noise figure of 3 dB' will have an equivalent noise source  $N_{ai}$  at its input, equal to the reference noise source of  $4.10^{-23}$ W/Hz. Indeed, if we look at figure 3, and make  $N_{ai}$  equal to  $N_{in}$ , the noise power doubles such that the SNR at the output will be 3 dB lower than the SNR at the input, and this is equal to the noise figure. This artificial noise source  $N_{ai}$  can now be thought of as fixed.

#### 1.2.2 Noise temperature

Because these very low noise powers are numerically difficult to deal with, the *effective noise* temperature  $T_{ai}$  (see also §1.1) of a noise source  $N_{ai}$  was defined as:

$$T_{ai} = (F-1).T_0 = (F-1).290K \tag{3}$$

When dealing with terrestrial links only, the noise figure F is the most interesting measure of performance of e.g. preamplifiers, because the incoming source temperature will typically be around 290K. In this case, the noise figure effectively indicates the loss in SNR. When dealing with satellite sources, the source temperature will generally be much lower, such that the noise figure becomes misleading. In this case, the effective noise temperature is more interesting, because it is independent of any reference. It is very important to note that the description of the noise figure as 'the difference in SNR between input and output' only holds if the source noise is indeed at 290K. When the source noise is much lower, as is often the case in space communications for instance, an amplifier 'having a noise figure of 3 dB' will degrade the SNR much more than 3 dB!

#### 1.2.3 Some examples

Some noise figures of interest are:

- a *lossy line* at temperature 290K has a noise figure equal to the insertion loss. In other words, if the line has a signal loss of 10 dB, the noise figure equals 10 dB as well.
- a *cascade of amplifiers*, e.g. an LNA and PAs (power amplifiers), such as in figure 4 has a composite noise figure equal to:

$$F_{cascade} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 \cdot G_2} \dots$$
(4)



Figure 4: amplifier cascade and their associated noise figure

From this it follows that the first amplifier stage is the most critical and should have the lowest noise figure  $(F_1)$ , and preferably high gain. Moreover, if the chain of amplifiers is preceded by a transmission line (as is most often the case), this transmission line can be regarded as an 'amplifier' with a gain lower than unity. Obviously such lines should be kept as short as possible, and preferably avoided (or placed *behind* the LNA), when dealing with very weak incoming signals!

Imagine one has to use a long cable, and wants to install a preamp to compensate for the transmission loss. Suppose the amplifier has G = 30 dB and F = 2 and the line has an insertion loss of 10 dB. The noise figure of the transmission line is equal to the insertion loss, as stated before, and equals 10 dB. The signal amplification of this system is equal to 100 (20 dB), regardless of the configuration. The noise, assuming the amplifier precedes the transmission line, equals

$$F_{\text{amp+line}} = 2 + \frac{10 - 1}{1000} = 2,009 \simeq 3 \text{ dB}.$$

If, on the other hand, the line precedes the amplifier, the noise figure is equal to

$$F_{line+amp} = 10 + \frac{2-1}{\frac{1}{10}} = 20 \simeq 13 \text{ dB}.$$

Quite a difference! In both cases, the signal amplification is equal to 20 dB, but the noise figure is 10 dB's apart.

## 2 Satellite communications

### 2.1 Atmospheric absorption

The Earth atmosphere extends about 20km above the Earth surface. Its best known effect is signal attenuation due to absorption by oxygen  $(O_2)$  and water  $(H_2O)$  molecules, fog and clouds. This absorption is very frequency dependent, as is shown in figure 5. Although it is not shown on this figure, obviously there will be a (small) increase in the absorption for lower elevations. Note the strong absorption peak due to water vapour at about 22 GHz.



Figure 5: Atmospheric absorption (from [1])

### 2.2 Antenna temperature

The temperature, associated with the noise at the terminals of an antenna, is referred to as the *antenna temperature*. As stated before ( $\S1.1$ ), the noise energy at the antenna output is a weighted average of the noise sources the antenna is looking at. The weighting factor is equal to the directivity towards the specific noise source. Imagine for example, that



Figure 6: Antenna temperature in the absence and presence of atmosphere

there is no atmosphere and the antenna looks towards space, as shown on the left in figure 6. In this case, the only noise contribution would be the cosmic noise, at about 3K. Of course, some stars (at high temperature) are always in view, but because they only occupy a very small angle in the antenna bundle, their contribution to the total noise is very low. Unfortunately, because the atmosphere absorbs energy, it also emits some noise of its own, as does every absorbing matter (shown on the right in figure 6). Luckily, it is far from a perfect absorber, such that it radiates less than a perfect black body would. Nonetheless, it contributes significantly in the total antenna temperature, and this contribution is referred to as *sky noise*. Because the sky noise is related to the absorption, it is also dependent on the frequency and the elevation the satellite is looking at. Therefore it comes as no surprise that the sky temperature chart in figure 7 looks very similar to the absorption curves of figure 5. In the extreme case that the elevation reaches  $0^{\circ}$ , the noise temperature becomes comparable to the Earth's ambient temperature. The concept of antenna temperature being the weighted average of different noise temperatures in view also shows an additional benefit



Figure 7: Sky temperature (from [2])

from the use of directional antennae. Indeed, directivity towards a signal source also implies a *reduced* sensitivity in other directions, e.g. the Sun. Man-made noise, emanating from machinery etc plays little role in frequencies above 1 GHz.

### 2.3 Effect of rain

The presence of rainfall has a dramatic influence on the link quality, due to absorption and reflection. Indeed, *rain scatter* for instance, however useful it may be for terrestrial links, suggests that a lot of signal doesn't arrive outside the atmosphere. The absorption is dependent on the rainfall intensity and on the path length. Figure 8 shows the specific attenuation due to rainfall and the effective path length. The combination of both graphs allows to calculate the attenuation.



Figure 8: Rainfall absorption: specific attenuation and effective path length [3]

Moreover, because of the high absorption, rain also acts as a rather strong noise source. To take this into account, the sky temperature from figure 7 has to be augmented with  $T_{rain}$ , equal to:

$$T_{rain} = T_m \left( 1 - \frac{1}{A_{rain}} \right), \tag{5}$$

with  $T_m$  the effective temperature of the rain (approximately 260K [3]), and  $A_{rain}$  the rainfall attenuation

### 2.4 System effective temperature

So far two noise sources have been identified: the antenna noise, being a weighted average of external noise sources, both natural and man-made, and the noise introduced by transmission lines and the preamplifier. The *system effective noise temperature* is equal to:

$$T_s = T_a + T_{rec},\tag{6}$$

with  $T_a$  the antenna temperature, and  $T_{rec}$  the receiver (line+preamp) temperature.

Obviously, low noise amplifiers cannot avoid antenna noise preceding them. Indeed, if a parabolic dish feed has a very wide angle (as in figure 9), the feed itself will not only 'look' towards the dish, but also to the environment behind the dish, at 290K! In satellite communications, it is therefore interesting to sacrifice some dish surface to avoid spillover. However, this results in a loss in both efficiency and gain. The tradeoff between gain and antenna noise is best reflected in the figure of merit

 $\frac{G}{T_s},$ 

also called *receiver sensitivity*.



Figure 9: Antenna noise sources. Note the dramatic effect of spillover

## 3 Example

A realistic link budget will now be evaluated for a satellite downlink at 24 GHz, such as e.g. the Amsat Oscar-40 K-band downlink [4]. Bright and rainy conditions are compared. We make the following assumptions:

- The transmitter has  $P_t = 30$  dBm signal power and the transmit antenna gain is  $G_t = 23$  dBi (the 'i' stands for the gain as compared to an isotropic antenna).
- The satellite is worked when it is at its apogee ('highest point'), at about 60000 km, with an elevation  $\theta = 45^{\circ}$ . Recall that the *free space loss*  $L_{fs}$ , i.e. the loss due to the divergence of the radio waves (without absorption) is calculated as:

$$L_{fs} = \frac{\lambda^2}{(4\pi d)},\tag{7}$$

with  $\lambda$  the wavelength in meters and d the distance in meters

At 24 GHz, this free space loss is equal to  $2, 7.10^{-22}$ , or -215 dB.

- From figure 5, one can read the atmospheric attenuation at 24 GHz as about 0, 5 dB under humid conditions.
- The specific attenuation of rain (assume 5mm/h) at 24 GHz can be read from figure 8 as about 1 dB/km. The effective path at an elevation  $\theta = 45^{\circ}$  is 5 km. This results in an attenuation of 5 dB.
- The increment in antenna noise temperature due to the rain can be calculated using formula 5 and is equal to  $T_{rain} = 178$  K.
- A parabolic dish with a diameter of 45 cm (and thus an area equal to  $0, 16m^2$ ) is used for reception, with an antenna efficiency of 50% (due to the low edge illumination to avoid spillover). The gain of such a dish is equal to:

$$G_r = \frac{4\eta \pi A_r}{\lambda^2},\tag{8}$$

with  $\eta$  the antenna efficiency,

 $A_r$  the (receive) antenna surface (in square meters), and  $\lambda$  the wavelength in meters.

For this example,  $G_r = 1, 3.10^4 = 41$  dB.

- The sky temperature under bright conditions can be read from figure 6 as about 30K.
- The receiver has a bandwidth of 3 kHz, and a noise figure equal to 3 dBm. According to equation 3, this corresponds to an equivalent noise temperature  $T_{rec}$  equal to 290K. For comparison, the link budget will also be calculated assuming a noise figure of 2 dB. This corresponds to a noise temperature  $T_{rec}$  equal to 170K.

	No rain		Rainfall	
	Signal	Noise	Signal	Noise
Transmitter power	30  dBm		30  dBm	
Transmit antenna gain	+23  dB		+23  dB	
Path loss	-215  dB		-215  dB	
Sky noise		30 K		$208 \mathrm{K}$
Atmospheric absorption	-0,5  dB		-0,5  dB	
Rainfall absorption			-5  dB	
Receive antenna gain	+41  dB		+41  dB	
Receiver noise temperature		290 K		290 K
System noise temperature		320 K		498 K
Total power in 3 kHz	-121,5 dBm	-138, 8  dBm	-126,5 dBm	-136,9 dBm

These figures have been collected in table 3. If no rain is present, the SNR is equal to 17,8 dB, which is very comfortable. Although rain only attenuates the signal by 5 dB, it lowers the SNR to 10,4 dB. Reception with the low-noise receiver (noise figure equal to 2dB) gives rise to SNRs that are respectively 19,7 dB and 11,5 dB. Note that the low-noise receiver is more beneficial in case of a bright sky (SNR improvement of 1,9 dB) than in case of rain (only 1,1 dB improvement).

# References

- [1] Recommendation ITU-R PI.676-2: Attenuation by atmospheric gases
- [2] Recommendation ITU-R PI.372-6: Radio noise
- [3] Satellite Communications Systems, G Maral & M Bousquet, Wiley 1987
- [4] http://www.amsat.org/amsat/sats/phase3d/k\_tx.html